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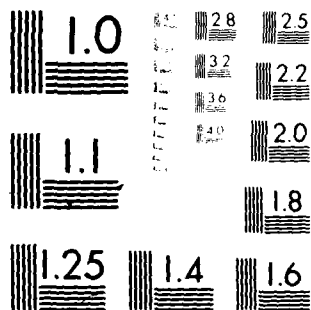UNCLASSIFIED   FTD-ID(RS)T-1107-81   NL

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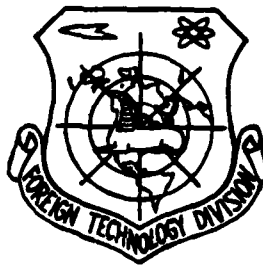MICROCOPY RESOLUTION TEST CHART
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ANTENNA

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U. S. BOARD ON GEOGRAPHIC NAMES TRANSLITERATION SYSTEM

Block	Italic	Transliteration	Block	Italic	Transliteration
А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

*ye initially, after vowels, and after ъ, ь; e elsewhere.
When written as ë in Russian, transliterate as yë or ë.

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	sinh⁻¹
cos	cos	ch	cosh	arc ch	cosh⁻¹
tg	tan	th	tanh	arc th	tanh⁻¹
ctg	cot	cth	coth	arc cth	coth⁻¹
sec	sec	sch	sech	arc sch	sech⁻¹
cosec	csc	csch	csch	arc csch	csch⁻¹

Russian	English
rot	curl
lg	log

1107,gw

SYNTHESIS OF A PLANE IMPEDANCE ANTENNA OF FINITE LENGTH

V. V. Chebyshev.

A method is examined for synthesis of an antenna in the form of
an impedance band in an unbounded conducting screen, excited by a
filament of magnetic current. An expression is obtained which joins
the radiation pattern of a very random type; this pattern is
described using odd Mathieu's functions, with a distribution of the
surface reactance in the limits of the band.

INTRODUCTION.

A large number of works have been dedicated to the problem of
synthesis of antennas. At the present time a rather full examination
has been made of the problems of calculation of the current of an
antenna with respect to a given radiation pattern. However, problems

of its modeling using certain structural elements have not been
sufficiently studied. Of known interest in this regard is the study
of impedance antennas for which the calculation of the surface
impedance, characterizing the field distribution in the antenna
aperture, may be connected with the calculation of its structural
elements.

For modeling the surface impedance it is possible to use a layer
of dielectric on a metal base or a ribbed structure; in actual
practice the use of the latter is preferable. Such a method of
construction is permissible under the condition of pure reactivity of
the impedance.

This significantly limits the class of radiation patterns
reproducible by an antenna with a purely reactive surface impedance.
The selection of this class of functions, approximating a given
radiation pattern during synthesis, is a very difficult problem the
solution of which is known only for patterns of a specific special
type [1], [2]. The purpose of this work is to develop a method of
synthesis of plane impedance antennas of finite length for more
random radiation patterns.

DERIVATION OF THE BASIC RELATIONSHIP

For the case of excitation of a TM-wave let us determine the
theoretical model of an impedance antenna in the form of an impedance
band -a$\le$y$\le$a (Fig. 1) in an unbounded conducting screen. The impedance
band is excited by an outside source in the form of a filament of
magnetic current V'. Such a model makes it possible to examine the
formation of an antenna radiation pattern in the plane of angle 6
depending on the one-dimensional distribution of impedance in the
limits of the band.

Fig. 1.

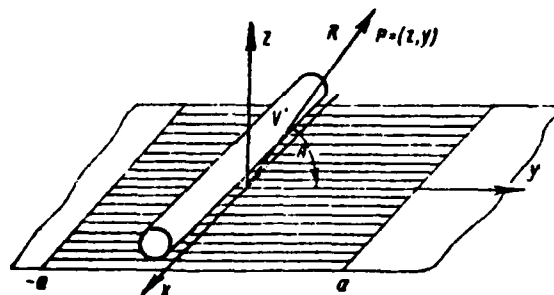In work [2] an expression was obtained which connects the
antenna radiation pattern with the distribution of surface impedance:

(1)
$$q(y) = \frac{\int_{-\infty}^{\infty} F(x) e^{-ixy}\,dx - 2\int_{-\infty}^{\infty} \text{ch}\,\gamma z_0 e^{-ix(y-y_0)}\,dx}{\int_{-\infty}^{\infty} \frac{F(x) e^{-ixy}}{\gamma}\,dx - 2\int_{-\infty}^{\infty} \frac{\text{ch}\,\gamma z_0 e^{-ix(y-y_0)}}{\gamma}\,dx},$$

where we accept the following designations

$x = \cos\theta; \quad \gamma = \sqrt{x^2 - 1}; \quad q(y) = -i\frac{Z(y)}{Z_0}; \quad Z_0 = \sqrt{\frac{\mu}{\varepsilon}}; \quad F(x)$ - function of the pattern;

$z = kz_1; \quad y = ky_1; \quad y_1 z_1$ - coordinates of Fig. 1; $\kappa = \frac{2\pi}{\lambda}; \quad y_0 z_0$ - coordinates of the current filament.

As is known, the case of pure reactive impedance is of greatest practical interest. The derivation of the condition of pure reactive impedance from (1) is connected with certain limitations imposed on the function of the radiation pattern and the position of the outside source, which in the general form is not possible. Therefore we shall limit ourselves to the case of assigning the outside source on the impedance surface z=0. Then from (1), using the definition of a δ-function, it is possible to obtain

$$(2) \qquad q(y) = \frac{\int\limits_{-\infty}^{\infty} F(x)\, e^{-ixy}\, dx}{\int\limits_{-\infty}^{\infty} \frac{F(x)\, e^{-ixy}}{\gamma}\, dx} + \delta(y - y_0).$$

where $\delta(y - y_0)$ - delta function, standardized relative to the denominator of the fraction in (2).

The effect of an outside source on the distribution of surface impedance is expressed by a characteristic in the form of a delta-function. The other term in (2) is determined by expressions for the density distribution of the electrical and magnetic currents on the impedance surface which do not depend on the position of the outside source. Therefore, if we assume the existence of the indicated characteristic in the distribution of the surface impedance then the examined problem is reduced to a uniform problem. In practice this designates the placement of the outside source in the area with sufficiently large values of the surface impedance. Therefore during derivation of the condition of pure reactivity of the impedance it is possible to limit ourselves to the examination of the first term in (2).

For the selected theoretical model of the antenna the function of the radiation pattern according to the conditions of the Wiener-Paley theorem must belong to the class W_h [3] where $h=\kappa a$ - is the size of the band. In this case the distribution function of the magnetic current $\phi(y)$ described by the numerator of the fraction (2) is finite in the limits of the band. During synthesis of an impedance antenna from class W_h one should select functions for which the condition of pure reactivity of impedance is satisfied.

For the numerator of fraction (2), by performing an inverse

Fourier transformation and by moving to the elliptical system of coordinates it is possible to obtain an integral Fredholm equation of the first type with a symmetrical nucleus, the solution of which may be expressed using eigenfunctions of the nucleus - odd Mathieu's functions [4].

Let us represent the radiation pattern in the form of an expansion with respect to Mathieu's functions:

$$(3) \qquad F(\theta) = \frac{h}{2\pi \sin \theta} \sum_m i^m b_m \, So_m (h, \cos \theta).$$

where b_m - actual coefficients,

So_m - odd Mathieu's functions according to the terminology of [6] which are represented by trigonometric expansions:

$$(4) \qquad \begin{aligned} So_{2m}(h, \cos\theta) &= \sum_{n-1} B_{2n, \, 2m} \sin 2n \, \theta \\ So_{2m+1}(h, \cos\theta) &= \sum_{h=0}^{\infty} B_{2n+1, \, 2m+1} \sin (2n + 1) \theta \end{aligned}$$

Then it is possible to obtain an expression for the synphase distribution of magnetic current on the impedance band:

$$\varphi(y) = i \sqrt{\frac{2}{\pi}} \left\{ \sum_m (-1)^m b_{2m} \frac{4 So'_{2m}(h, 0)}{h B_{2,2m}} So_{2m}(h, y/h) - \right.$$

(5)

$$\left. + \sum_m (-1)^m b_{2m+1} \frac{2 So_{2m+1}(h, 0)}{B_{1,2m+1}} So_{2m+1}(h, y/h) \right\}.$$

Let us determine the distribution of electrical current which can be calculated with the substitution of the function of the pattern (3) into the integral expression of the denominator of the fraction (2).

For this let us examine the indicated expression with a change of X in the section $[-1, 1]$ which corresponds to the change of angle $\theta[0, \pi]$. Let us note that in the expansion (3) Mathieu's functions of an even order $2m$ are odd functions relative to the angle $\theta = 0.5\pi$, and of an odd order $(2m+1)$ - even. Therefore, for example, for an even function of the pattern $F_u(\theta)$ we have

(6)

$$\int_{-1}^{1} \frac{F_u(\theta) e^{-iy} \, dx}{\gamma} = \frac{h}{2\pi} \sum_m (-1)^m b_{2m+1} \times$$

$$\times \int_0^{\pi} \frac{So_{2m+1}(h, \cos\theta) \cos(y \cos\theta) \, d\theta}{\sin\theta}.$$

It is known that:

$$\cos(y \cos \theta) = J_0(y) + 2 \sum_{n=1}^{\infty} (-1)^n J_{2n}(y) \cos 2n\,\theta.$$

where $J_m(y)$ - is a Bassel function.

Using the latter relaticnship and the representation of
Mathieu's functions (4), calculation of expression (6) may be reduced
to calculation of tabular integrals of combinations of trigonometric
functions which , as follows frcm the relationship in [5], are equal
to zero. This conclusicn is valid also for the case of an odd
function of the pattern. Consequently the distribution of the
electrical current on the impedance plane is determined by the
integral expression of the dercminater of the fraction (2) with
values $\varkappa > 1$.

The range of values $\varkappa > 1$ ccrresponds to the range of imaginary
angles $\theta = i\alpha$ for $\varkappa = Ch\alpha$, where $\alpha > 0$.

Then

$$i\,\psi_m(y) = i\sum_m (-1)^{m+1} \cdot b_{2m} \int_0^\infty \frac{So_{2m}(h . \, Ch\,\mu)\sin(y\,Ch\,\mu)\,d\mu}{Sh\,\mu} +$$

(7)

$$+ i\sum_m (-1)^m b_{2m+1} \int_0^\infty \frac{So_{2m+1}(h . \, Ch\,\mu)\cos(y\,Ch\,\mu)\,d\mu}{Sh\,\mu} .$$

where $Ch\,\mu$, $Sh\,\mu$ - hyperbolic functions.

The convergence of series (7) follows from the uniform or root-mean-square convergence of series (3) to the assigned function of the pattern for $[0, \pi]$.

Mathieu's functions are tabulated for values of angles $\theta[0, \pi]$. In the range of imaginary angles, i.e. for $\varkappa > 1$, Mathieu's functions may be calculated from the relationships in [6]:

$$(8) \quad \begin{aligned} So_{2m}(h, \, Ch\mu) &= -\frac{4Cth\mu}{h^2 B_{2,2m}} \sum_{n=1}^{\infty} B_{2n, \, 2m} J_{2n}(hSh\mu) \\ So_{2m+1}(h, \, Ch\mu) &= \frac{2}{B_{1, \, 2m+1}} \sum_{n=0}^{\infty} B_{2n+1, \, 2m+1} J_{2n+1}(hSh\mu) \end{aligned}$$

where J_m - Bessel function, $B_{2n, 2m}, B_{2n+1, 2m+1}$ - coefficients of expansions of (4).

Series (8) converge rather well and permit limitation of the number of terms.

During calculation of integral expressions in (7) taking into account (8) it is necessary to use methods of numerical integration. If the value of the upper limit does not exceed 4, then for Simpson's method the error of calculations is less than 10% for $h=10$ and will be less, the greater the value of h.

The distributions of the magnetic (5) and electric (7) currents have a synphase character and satisfy the condition of pure reactivity for surface impedance. Let us substitute (5) and (7) in (2) and obtain

(9)
$$q(y) = 1 \ 2\pi \ \frac{\sum\limits_{m} (-1)^{m+1} b_{2m} \dfrac{4So'_{2m}(h,\ 0)}{h^2 B_{2,\ 2m}} So_{2m}(h,\ y/h) \times \rightarrow}{\Psi_{2m}(y) + \Psi_{2m+1}(y)}$$
$$\rightarrow \times \sum\limits_{m} (-1)^{m} b_{2m+1} \frac{2So_{2m+1}(h,\ 0)}{hB_{1,\ 2m+1}} So_{2m+1}(h,\ y/h)$$

Expression (9) makes it possible to determine the distribution of surface impedance in the limits of a band with a size of 2h for the function of pattern (3).

Let us note that the radiation pattern of the examined type has an even amplitude and an odd phase characteristic, i.e., it is symmetrical relative to the origin Fig. 1. Therefore, during assignment of the radiation pattern in (3) it is possible to limit oneself to Mathieu's functions of even or odd orders.

Let us determine the position of the filament of magnetic current on the impedance band. Its effect on the distribution of the surface impedance is expressed by a characteristic in the form of a delta function in (2) which was previously excluded from examination.

On the strength of the first boundary problem of electrodynamics the emission of the impedance antenna may be considered as the emission of a magnetic current which is finite in the limits of the band. For the radiation pattern (3) described by Mathieu's functions of even and odd orders the impendance antenna has a phase center. In this case the distribution of magnetic current in the band must be symmetrical [3] and the filament of the magnetic current should be located in the middle of the band.

ASSIGNMENT OF THE RADIATION PATTERN

During synthesis of an impedance antenna the function of the radiation pattern may be assigned by a set of Mathieu's harmonics of even or odd orders in (3) according to the method of partial patterns which is known from the theory of synthesis of linear antennas.

Fig. 2 shows the theoretical and experimental radiation patterns of symmetrical form

$$F(\theta) = \frac{1}{\sin\theta} \, [So_3(h, \cos\theta) + So_5(h, \cos\theta)]$$

for the band h=10. The distribution of the surface impedance, calculated according to formula (9), is given in Fig. 3. The surface impedance is modeled by a ribbed structure with a period of 0.05 λ

placed in a screen 10λ with $\lambda=10\,cm$. The outside source has the form of a narrow radiating slot which is placed in the middle of the band between two channels of the ribbed structure with a depth of 0.23λ (area of the source, Fig. 3).
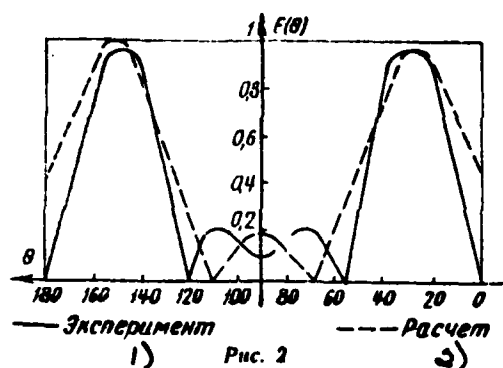
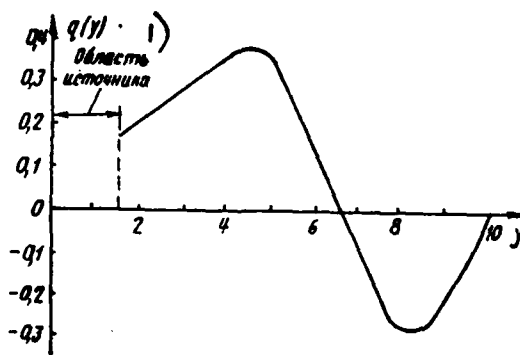Fig. 2.   KEY: 1. experiment; 2. theory.

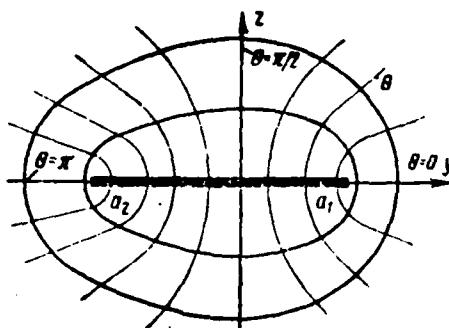Fig. 3.   KEY: 1. area of the source.

Fig. 4.

From Fig. 2 it is evident that the coincidence of the
theoretical and experimental radiation patterns is rather good. The
examined method may also be used for the case of an asymmetric
radiation pattern. This case is connected with the selection of the
position of the outside source which is displaced relative to the
middle of the impedance band in the following manner. The conclusion
that for a symmetrical radiation pattern the outside source should be
located in the middle of the band can be examined as a consequence of
the condition of pure reactivity which is satisfied for the surface
impedance in (9) with a certain selection of coefficients in the
expansion with respect to Mathieu's functions (3). The latter are
eigenfunctions of the problem of excitation of the impedance band in

an elliptical system of coordinates).

Let us suppose that the outside source is displaced relative to the middle of the band and represent the half-space, limited by plane z=0, in the form of two quadrants for angles $0 \leqslant \theta \leqslant 0.5\pi$ and $0.5 \leqslant \theta \leqslant \pi$ in elliptical coordinates Fig. 4.

Let us assign the radiation pattern in the indicated quadrants by expansions of the type (3) for parameters h_1 and h_2, corresponding to dimensions of the half-bands a_1 and a_2 Fig. 4, where $a_1 + a_2 = 2a$, and let us examine the conditions to which these expansions must correspond.

The function of the pattern belongs to class W_A, i.e., it is an analytical function. Let us consider the functions describing the radiation pattern in various quadrants by analytical continuation one after another, and "join" them on the boundary of quadrants by the condition

$$(10) \qquad \begin{matrix} F_1(\theta) = F_2(\theta) \\ F_1'(\theta) = F_2'(\theta) \end{matrix} \Bigg| ,$$

where $F_1(\theta)$, $F_2(\theta)$ - functions describing the radiation pattern in various quadrants.

Condition (10) will be all the more valid for determination of the analytical continuation of the function of the pattern, the closer the values of the higher derivatives of functions $F_1(\theta)$ and $F_2(\theta)$ are to zero with $\theta=0.5\pi$. Therefore condition (10) may be recommended for Mathieu's functions of an odd order $(2m+1)$ in (3) with not too large values of the latter. Calculation of the distribution of the surface impedance is carried out for each selected half-band on both sides of the exciting source in analogy with (9).

The method of partial patterns is distinguished by graphicness. However, for highly directed patterns it is connected with a large volume of calculations. Therefore the method suggested in [7] for assigning a pattern may turn out to be more convenient. The radiation pattern $F(\theta)$ assigned in the section $\theta[0, 2\pi]$ by a function of class L_2 * is represented by a Fourier series which then, on the basis of expansion of Mathieu's functions (4), is expressed by the series (3).

[FOOTNOTE: *Quadratic integrands belong to class L_2. END FOOTNOTE].

For example, for an even function of the pattern the coefficients of this series have the form

(11)
$$b_{2m+1} = \sum_{n=1}^{\infty} B_{2n+1,\,2m+1}\, a_{2n+1},$$

where
$$a_{2n+1} = \frac{1}{M_{2n+1}} \int_{0}^{\pi} F(\theta) \sin\theta \sin(2n+1)\theta\, d\theta;$$

$B_{2n+1,\,2m+1}$ – coefficients of expansion (4):

M_{2m+1} – norm of Mathieu's functions of an odd order [6].

Similarly for an odd function of a pattern the coefficients of the series are determined using Mathieu's functions of an even order. Series (11) converges. Actually, the coefficients $B_{2n+1,\,2m+1}$ and a_{2n+1} are coefficients of the Fourier expansion of the Mathieu function and of the radiation pattern with respect to trigonometric functions which form a complete system in class L_2. Therefore formula (11) may be considered as an analog of the generalized formula of Parseval for the product of two functions of class L_2.

From the latter, taking into account the limitedness of the norm of functions in this class, convergence of series (11) also follows.

Continuous functions can also be assigned to class L_2. This makes it possible to assign a radiation pattern $P(\theta)$ by a continuous function, which is convenient in practice.

Let us examine the case of assignment of a radiation pattern in the section $\theta[0, \pi]$, which determines the range of emission of the antenna. Let us represent the radiation pattern of a random type by the sum of even and odd functions relative to the angle $\theta=0.5\pi$, which can always be done. It is known from [8] that for any function, continuous on section $[0, \pi]$, there exists a trigonometric polynomial of the best approximation of the type

(12)
$$P(\theta) = \sum_{\kappa=0}^{\infty} C_\kappa \cos \kappa\, \theta.$$

For even values 2k in (12) the function $P(\theta)$ is even, for odd values (2k+1) - odd.

Determining the polynomial (12) for the assigned radiation pattern according to the method of [8] from relationship (11) it is possible to determine the coefficients of expansion of the latter with respect to Mathieu's harmonics in (3). The examined method of assignment of the pattern may turn out to be useful when using digital computers for calculations.

Limitation of the number of terms of the series (3) leads to an approximate reproduction of the assigned pattern which may be evaluated as in [7]. Depending on the number of terms in (3), the width of the radiation pattern, and the value of the impedance band it is possible to obtain ultradirectional solutions which are expressed in the abruptly oscillating character of distribution of the surface impedance of an unmodeled ribbed structure. Therefore during synthesis of an impedance antenna a series of calculations should be made for various values h, for the purpose of selecting a suitable version.

CONCLUSIONS

The suggested method of synthesis of a plane impedance antenna of finite length permits assignment of symmetrical radiation patterns in coordinates Fig. 1. This expands the possibilities of using impedance antennas when designing antenna systems.

The condition of pure reactivity of the surface impedance is expressed in a certain selection of coefficients in the expansion of the pattern according to odd Mathieu's functions (3) and in the selection of the position of the outside source - of the filament of

magnetic current on the impedance band. With a symmetrical radiation
pattern the filament of magnetic current is located in the middle of
the impedance band. In the case of an asymmetrical radiation pattern
the position of the filament depends on the selection of functions of
the pattern in two quadrants of the half-space in the framework of
condition (10) which imposes known limitations on the type of the
modeled pattern.

Among the shortcomings of the examined method are the numerical
determination of integral expressions in the denominator of the
fraction (2) which determines the surface impedance. This difficulty
can be avoided when making up standard tables for a corresponding set
of Mathieu's functions describing radiation patterns (3) with various
dimensions of the impedance band. Existing tables of Mathieu's
functions make it possible to design impedance antennas with a band
$2a \leqslant 3\lambda$ which limits the use of antennas of this type as highly
directional antennas. Therefore the necessity should be recognized of
making up tables of Mathieu's functions for large bands.

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EMISSION OF AN ELEMENTARY SLOT VIBRATOR LOCATED IN THE CENTER OF AN
IDEALLY CONDUCTING DISK

Yu. V. Pimenov, L. G. Braude.

On the basis of the solution of a strict integral equation we
obtained asymptotic expressions for the field arising in the far zone
during excitation of an ideally conducting disk by an elementary slot
vibrator (magnetic dipole) located in the center of the disk. During
solution it was assumed that the radius of the disk is much greater
than the wavelength.

INTRODUCTION

The emission of an elementary slot vibrator, located in the
center of an infinitely thin ideally conducting round disk, was
examined by M. G. Belkina in work [1]. A solution was obtained on the
basis of the Fourier method in the form of series with respect to
spheroidal functions. As is known, such series in the case of a disk,
which is large in comparison with the wavelength, converge extremely
slowly and the solution becomes practically unsuitable for numerical

calculations. Therefore it is of interest to obtain an asymptotic

solution for the case ka>>1 where k=2π/λ - wave number; λ -

wavelength; a - radius of the disk.

A slot cut in one side of the disk is equivalent to an

elementary magnetic vibrator lying in the disk. On the strength of

the principle of duality [2] it is possible, instead of the problem

of excitation of a disk by an elementary magnetic vibrator located in

the center of the disk, to solve the problem of excitation of an

ideally conducting plane with a round opening by an elementary

electrical vibrator located in the center of the opening, and then

according to known formulas of the transition, to find the solution

of the initial problem. With ka>>1 the second (auxiliary) problem is

solved considerably more simply.

STATEMENT OF THE PROBLEM

Let us examine the auxiliary problem of excitation of an ideally

conducting plane with a round opening of radius a by an elementary

electrical vibrator with moment $\vec{p}$, located in the center of the

opening.

Let us introduce a Cartesian coordinate system x, y, z, the

origin of which coincides with the center of the opening, axis z is

perpendicular to the plane of the screen and the direction of axis x
coincides with the direction of the moment of the vibrator $(\vec{p}=\vec{x}_0 p)$.
Let us also introduce a cylindrical system of coordinates r, ϕ, z,
axis z of which coincides with axis z of the Cartesian coordinate
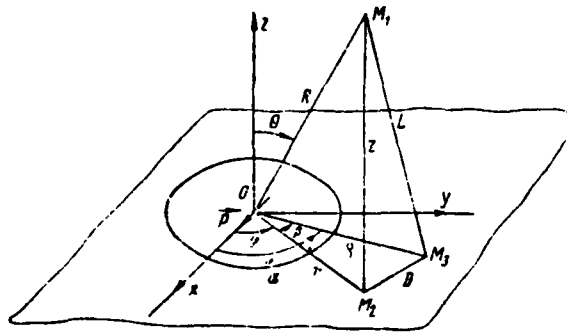system (Fig. 1).

Fig. 1.

The intensity of the primary electrical field created by the
elementary electrical vibrator

(1) $\vec{E}_1 - \vec{r}_0 E_{1r} + \vec{\phi}_0 E_{1\phi} + \vec{z}_0 E_{1z}$

where

$$E_{1r} = f_1(r,\ z)\cos\varphi;\ E'_{1\varphi} = f_2(r,\ z)\sin\varphi;\ E_{1z} = f_3(r,\ z)\cos\varphi;$$

$$f_1(r,\ z) = M\,\frac{e^{-i\kappa R}}{R^3}\left[z^2 - \frac{i}{\kappa R}\left(1 - \frac{i}{\kappa R}\right)(z^2 - 2r^2)\right];$$

$$f_2(r,\ z) = - M\,\frac{e^{-i\kappa R}}{R}\left[1 - \frac{i}{\kappa R} - \frac{1}{(\kappa R)^2}\right];$$

$$f_3(r,\ z) = - M\,\frac{e^{-i\kappa R}}{R}\,\frac{r}{R}\left[1 - \frac{3i}{\kappa R} - \frac{3}{(\kappa R)^2}\right]\frac{z}{R};$$

$$M = \frac{p\kappa^2}{4\pi\varepsilon};\ R = \sqrt{r^2 + z^2}.$$

and $\mathcal{E}$ - dielectric constant cf the medium. The dependence on time is taken in the form $e^{i\omega t}$.

Under the effect of the field (1) on a plane with a round opening currents are induced with a density

(2) $$\vec{j}(r,\ \varphi) = \vec{r}_0 j_r(r,\ \varphi) + \vec{\varphi}_0 j_\varphi\ (r,\ \varphi) = \vec{x}_0 j_x(r,\ \varphi) + \vec{y}_0 j_y(r,\ \varphi).$$

The vector potential corresponding to these currents

(3) $$\vec{A} = \frac{\mu}{4\pi}\int_a^{\infty}\rho\,d\rho\int_{\varphi}^{2\pi+\varphi}\frac{e^{-i\kappa L}}{L}\,j(\rho,\ \varkappa)\,d\varkappa.$$

where

$$L = \sqrt{r^2 + \rho^2 + z^2 - 2r\rho\cos(\varkappa - \varphi)}.$$

and μ - magnetic permeability of the medium. G. A. Grinberg showed (see [3] or [4]), that in the case of ideally conducting infinitely thin screens the vector potential $\vec{A}$ in points of the screen may be found independently of the function $\vec{j}(r, \phi)$. This makes it possible, applying relationship (3) to points of the screen, to reduce the problem to the solution of an integral equation of the first type. For determination of the function $\vec{A}$ on the screen, i.e., with r>a, z=0, we proceed in the following manner.

The intensity of the secondary electrical field $\overleftarrow{E_1}$ is connected with the vector potential $\vec{A}$ by the relationship

(4)
$$\vec{E}_1 = -\operatorname{grad} \Psi - i\omega \vec{A},$$

where
$$\Psi = \frac{1}{i\omega\varepsilon\mu}\operatorname{div}\vec{A}.$$

On the surface of the screen the following boundary conditions must be satisfied:

(5)
$$E_{1r} = -E_{1r}' \quad \text{при } r \geqslant a, \ z = 0;$$
$$E_{1\varphi} = -E_{1\varphi}' \quad \text{при } r > a, \ z = 0.$$
(6)

which, taking (4) into account, can be rewritten in the form

(7)
$$\frac{\partial \Psi}{\partial r} + i\omega A_r = E'_{1r}, \text{ при } r \geqslant a, z = 0;$$

(8)
$$\frac{1}{r}\frac{\partial \Psi}{\partial \varphi} + i\omega A_\varphi = E'_{1\varphi}, \text{ при } r \geqslant a, z = 0,$$

where A_r and A_φ - respectively are the radial and azimuthal components of vector $\vec{A}$.

Since $E'_{1r} = f_1(r, z)\cos\varphi$, and $E'_{1\varphi} = f_2(r, z)\sin\varphi$, then, according to the results of work [3], the scalar potential Ψ on the surface of the screen may be represented in the form

(9)
$$\Psi = \psi(r)\cos\varphi \text{ при } r > a, z = 0,$$

whereby the function $\psi(r)$ must satisfy the condition of emission and the differential equation

(10)
$$\frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r}\frac{d\psi}{dr} + \left(\kappa^2 - \frac{1}{r^2}\right)\psi = -\frac{\partial f_2}{\partial z}\bigg|_{z=0}; \ r > a.$$

Solving (10) using the method of variation of random constants and taking into account the condition of emission, we obtain

$$\psi(r) = BH_1^{(2)}(\kappa r) + \frac{\pi i}{4} \left\{ H_1^{(1)}(\kappa r) \int_r^{\tilde{}} F(l) H_1^{(2)}(\kappa l) dl - \right.$$

$$\left. - H_1^{(2)}(\kappa r) \int_r^{\tilde{}} F(l) H_1^{(1)}(\kappa l) dl \right\}, \quad r \sim a,$$

(11)

where

$$F(l) = - M \frac{e^{-i\kappa l}}{l^2} \left[1 - \frac{3i}{\kappa l} - \frac{3}{(\kappa l)^2} \right].$$

$H_1^{(1)}$ and $H_1^{(2)}$ - Hankel functions of the first order, first and second types respectively, and B - a certain constant which must be determined subsequently from the condition of reduction of the radial component of current density to zero on the edge of the opening:

(12) $j_r(a) = 0.$

Thus, function $\psi(r)$, and consequently, also function $\Psi(r, z)$ with $r > a$; z=0 are determined with an accuracy up to constant B. Expressing components A_r and A_φ from (7) and (8) and moving then to the Cartesian components of vector $\vec{A}$, we obtain

(13) $\vec{A} = \vec{x}_0 A_x + \vec{y}_0 A_y.$

where

(14)
$$A_x = A_x^{(0)}(r) + A_x^{(2)}(r) \cos 2\varphi \quad \text{при} \quad r > a; \ z = 0$$
$$A_y = A_y^{(2)}(r) \sin 2\varphi \quad \text{при} \quad r \quad a; \ z \quad 0$$

$$(15) \quad \begin{aligned} A_z^{(0)}(r) &= \frac{i}{2\omega}\left[\frac{d\psi}{dr} - f_1(r,\ 0) + \frac{1}{r}\psi + f_2(r,\ 0)\right] \\ A_x^{(2)}(r) &= A_y^{(2)}(r) = \frac{1}{2\omega}\left[\frac{d\psi}{dr} - f_1(r,\ 0) - \frac{1}{r}\psi - f_2(r,\ 0)\right] \end{aligned} \Bigg\} .$$

Applying (3) to points of the screen ($r > a$, $z=0$) and taking into account (13) we arrive at two independent integral equations of the first type:

$$(16) \quad A_z^{(0)}(r,\ 0) + A_z^{(2)}(r,\ 0)\cos 2\varphi = \frac{\mu}{4\pi}\int\limits_a^\infty \rho\, d\rho \int\limits_\varphi^{\varphi+2\pi} \frac{e^{-i\kappa D}}{D} j_x(\rho,\ \alpha)\, d\alpha;$$

$$(17) \quad A_y^{(2)}(r,\ 0)\sin 2\varphi = \frac{\mu}{4\pi}\int\limits_a^\infty \rho\, d\rho \int\limits_\varphi^{\varphi+2\pi} \frac{e^{-i\kappa D}}{D} j_y(\rho,\ \alpha)\, d\alpha.$$

where
$$D = \sqrt{r^2 + \rho^2 - 2r\rho\cos(\alpha - \varphi)}.$$

From the form of the left parts of equations (16), (17) it follows that the functions $j_x(\rho,\ \alpha)$ and $j_y(\rho,\ \alpha)$ may be sought in the form

$$(18) \quad \begin{aligned} j_x &= j_x^{(0)}(\rho) + j_x^{(2)}(\rho)\cos 2\alpha \\ j_y &= j_y^{(2)}(\rho)\sin 2\alpha \end{aligned} \Bigg\} .$$

in which case $\quad j_y^{(2)}(\rho = j_x^2 \rho)$.

Substituting (18) into (16) and moving in the internal interval to a new variable of integration β according to the formula $\beta = \alpha - \phi$ we arrive at two independent integral equations of the first type for functions $j_x^{(0)}(\rho)$ and $j_x^{(2)}(\rho)$:

$$(19) \qquad A_x^{(\nu)}(r, 0) = \frac{\mu}{4\pi} \int_a^\infty j_x^{(\nu)}(\rho)\rho\, d\rho \int_0^{2\pi} \frac{e^{-\kappa D}}{D} \cos \nu\beta\, d\beta; \quad r \geqslant 0; \quad \nu = 0; 2.$$

where $\qquad D = \sqrt{r^2 + \rho^2 - 2r\rho\cos\beta}.$

The left parts of equations (19) are known with an accuracy up to constant B, which must be determined after finding the functions $j_x^{(0)}(\rho)$ and $j_x^{(2)}(\rho)$. For calculating constant B we use the condition (12) which, after transition to functions $j_x^{(0)}(\rho)$ and $j_x^{(2)}(\rho)$ acquires the form

$$(20) \qquad j_x^{(0)}(a) + j_r^{(2)}(a) = 0.$$

Thus, the problem of excitation of an ideally conducting plane with a round opening by an elementary electrical vibrator located in the center of the opening is reduced to the solution of two independent integral equations of the first type (19) with a

supplementary condition (20).

DETERMINATION OF CURRENTS

Equations (19) are strict integral equations of the problem.
They are valid with any values of the parameter ka. We are interested
in the solution of these equations with ka>>1. In this case the left
parts of equations (19), determined by formulas (15) are simplified
considerably. Since ka>>1, and r>a, the Hankel functions entering the
left parts of equations (19) may be replaced by the first terms of
their asymptotic expansions:

$$(21) \qquad H_v^{(2)}(\kappa r) \approx \frac{\sqrt{2}}{\sqrt{\pi \kappa r}} e^{-i\kappa r} e^{i\left(\frac{\tau}{4} - \frac{v\tau}{2}\right)}.$$

Disregarding, in addition, terms of the order 1/kr in comparison with
unity, we obtain

$$(22) \qquad \omega p \left| B' \frac{e^{-i\kappa r}}{\sqrt{r}} - \delta_v i \frac{e^{-i\kappa r}}{\sqrt{r}} \right| =$$
$$\int_a^\infty j_x^{(v)}(\rho) \rho \, d\rho \int_0^{2\pi} \frac{e^{-i\kappa D}}{D} \cos \beta \, d\beta; \quad r \geqslant a; \quad v = 0; \ 2.$$

where

$$B' = B \frac{2}{\rho \kappa} \left| \overline{\frac{2\pi}{\kappa}} \right| \frac{e^{i\frac{3\pi}{4}}}{\kappa} \varepsilon; \quad \delta_\nu = \begin{cases} 1 \\ 0 \end{cases} \text{ where } \begin{matrix} \nu = 0, \\ \nu = 2. \end{matrix}$$

The internal integral in the right part of (22) may be transformed using the asymptotic equality proved in [5]:

(23)
$$\int_0^{2\pi} \frac{e^{-i\kappa D}}{D} \cos \nu\beta \, d\beta = -\frac{\pi i}{\sqrt{r\rho}} \{ H_0^{(2)}(\kappa|r-\rho|) + $$
$$+ i(-1)^\nu H_0^{(2)}[\kappa(r+\rho)]\} + O\left[(\kappa a)^{-3/2}\right], \quad \nu = 0; 2.$$

Substituting (23) into (22) and introducing the dimensionless variables ξ, η, and γ, which are connected with ρ, r, and k by relationships $\rho = a(1+\xi), \; r = a(1+\eta), \; \gamma = \kappa a,$ (24)

we obtain

(25)
$$\int_0^\infty u^{(\nu)}(\xi) H_0^{(2)}(\gamma|\eta-\xi|) d\xi + i \int_0^\infty u^{(\nu)}(\xi) H_0^{(2)}[\gamma(\eta+\xi+2)] d\xi = $$

where
$$= C e^{i\frac{\pi}{4}} \frac{\sqrt{2}}{\sqrt{\pi\gamma}} e^{-i\gamma\eta} - \delta_\nu H_0^{(2)}[\gamma(\eta+1)],$$

(26)
$$u^{(\nu)}(\xi) = \frac{\sqrt{2\pi}}{i\omega\sqrt{\gamma\rho}} e^{-i\frac{\pi}{4}} a^2 \, j_x^{(\nu)}[a(1+\xi)] \sqrt{1+\xi}, \quad \nu = 0; 2,$$

and $C = -i\sqrt{a} \, e^{-i\gamma} B'$ - a certain constant which will subsequently be determined from condition (20).

Using the equalities proved by G. A. Grinberg [6]

(27)
$$\int_0^\infty \frac{e^{-i\gamma(\xi+R_1)}}{\pi\sqrt{\xi}(\xi+R_1)} \sqrt{R_1} \, H_0^{(2)}(\gamma|\eta-\xi|) d\xi = H_0^{(2)}[\gamma(\eta+R_1)].$$

(28)
$$\int_0^\infty \frac{\sqrt{\gamma}\, e^{-i\frac{\pi}{4}}}{\sqrt{2\pi\xi}} \cdot e^{-i\gamma\xi} H_0^{(2)}(\gamma|\eta-\xi|)\,d\xi = e^{-i\gamma\eta}.$$

we transform equations (25) into integral equations of the second
type:

$$u^{(\nu)}(\xi) = -i\,\frac{e^{-i\gamma(\xi+2)}}{\pi\sqrt{\xi}} \int_0^\infty \frac{u^{(\nu)}(t)\,e^{-i\gamma t}\sqrt{t+2}}{\xi+t+2}\,dt \;+$$
(29)
$$+\,C\frac{e^{-i\gamma\xi}}{\pi\sqrt{\xi}} - \delta_\nu\,\frac{e^{-i\gamma(\xi+1)}}{\pi\sqrt{\xi}(\xi+1)}, \quad \nu=0;\ 2.$$

Since according to the supposition that $\gamma = ka \gg 1$, the solution of
equations (29) may be found by the method of successive
approximations. However, it is more convenient to use an artificial
method.

Functions $u^{(\nu)}(\xi)$ are proportional to components $j^{(\nu)}[a(1+\xi)]$ of
current density induced in the screen. With an increase in variable ξ
functions $u^{(\nu)}(\xi)$ decrease in absolute value and approach zero with
$\xi \to \infty$. Therefore with $\gamma \gg 1$ the basic contribution to the value of the
integral, entering into (29), is made by the vicinity of the point
$\xi = 0$. Consequently, the following approximate equality occurs:

(30)
$$u^{(\nu)}(\xi) = -i \cdot \frac{\sqrt{2}}{\pi} \cdot \frac{e^{-i\gamma(\xi+2)}}{\sqrt{\xi}(\xi+2)} \, U_0^{(\nu)} + C \, \frac{e^{-i\gamma\xi}}{\pi \sqrt{\xi}} -$$
$$- \delta_\nu \frac{e^{-i\gamma(\xi+1)}}{\pi(\xi+1)}, \quad \nu = 0; \, 2.$$

where

(31)
$$U_0^{(\nu)} = \int_0^\infty u^{(\nu)}(\xi) e^{-i\gamma\xi} d\xi, \quad \nu = 0; \, 2.$$

It may be strictly shown that the error of equation (30) does not exceed $O(\gamma^{-3/2})$.

For determination of constants $U_0^{(\nu)}$ we multiply both parts of (30) by $e^{-i\gamma\xi}$ and integrate with respect to ξ from zero to infinity. As a result we arrive at two (for $\nu = 0$ and $\nu = 2$) independent algebraic equations, solving which, we obtain

(32)
$$U_0^{(\nu)} = \frac{\dfrac{C e^{i\frac{\pi}{4}}}{\sqrt{2}\sqrt{\pi\gamma}} - \delta_\nu [1 - \Phi(\sqrt{i 2\gamma})]}{1 + i e^{i2\gamma} [1 - \Phi(\sqrt{i 4\gamma})]}, \quad \nu = 0; \, 2.$$

where

(33)
$$\Phi(\sqrt{i w}) = \frac{2 e^{i\frac{\pi}{4}}}{\sqrt{\pi}} \int_0^{\sqrt{w}} e^{-is^2} ds.$$

Constant C remains to be determined. Using condition (20) which following transition to functions $u^{(\nu)}(\xi)$ acquires the form

(34) $\dot{u}^{(0)}(0) + u^{(2)}(0) = 0,$

we obtain

(35) $C = \dfrac{e^{-iv}}{2} \cdot \dfrac{1 + i\,e^{i2v}\,[1 - \Phi(\sqrt{i4\gamma})] - \dfrac{i}{\sqrt{2}}\,[1 - \Phi(\sqrt{i2\gamma})]}{1 + i\,e^{i2v}\,[1 - \Phi(\sqrt{i4\gamma})] - i\,\dfrac{e^{-i2v}\,e^{-ir/4}}{2\sqrt{\pi\gamma}}}.$

Expression (35) is considerably simplified if the function $\Phi(\sqrt{i\omega})$ is replced by its asymptotic distribution. In this case the simple relationship $C = \dfrac{1}{2} \cdot e^{-iv} + O(\gamma^{-3/2}).$ will be satisfied.

Thus, functions $u^{(v)}(\xi)$ are completely determined and consequently the distribution of currents induced on the screen is known.

DETERMINATION OF THE FIELD

Let us move to determination of the field arising during excitation of an ideally conducting plane with a round opening by an elementary electrical vibrator located in the center of the opening.

The vector potential of currents induced on the screen is

expressed by formula (3) and has two components A_x and A_y, in which case

(36)
$$A_x = A_x^{(0)}(r,\ z) + A_x^{(2)}(r,\ z)\cos 2\varphi$$
$$A_y = A_y^{(2)}(r,\ z)\sin 2\varphi;\ \ A_y^{(2)} = A_x^{(2)}$$

On the screen ($r \geqslant a$, z=0) functions $A_{ix}^{(\nu)}(r,\ z)$ coincide with the functions $A_x^{(\nu)}(r)$, introduced earlier, and in a random point of space are determined by the expression

(37)
$$A_x^{(\nu)} = \frac{\mu}{4\pi} \int\limits_a^\infty j_x^{(\nu)}(\rho)\rho\, d\rho \int\limits_0^{2\pi} \frac{e^{-i\kappa L}}{L}\cos\nu\beta\, d\beta,\ \ \nu = 0;\ 2.$$

Formula (37) is not convenient for numerical calculations. Let us find its asymptotic distribution. In this case we shall consider two ranges: the first - adjacent to axis z, the second - the remaining part of space.

First let us examine the second range. Let us introduce the spherical system of coordinates R, θ, ϕ, the polar axis of which coincides with axis z of the cylindrical system of coordinates (Fig. 1).

Formula (37) in this system of coordinates acquires the form

(38)
$$\Lambda_x^{(\nu)} = \frac{i e^{-i\frac{\pi}{4}} \omega \mu \rho I}{4\pi} \frac{\gamma}{2\pi a} \int_0^{\infty} u^{(\nu)}(\xi) \sqrt{1 + \xi}\, d\xi \int_0^{2\pi} \frac{e^{-i\gamma_0 L_0}}{L_0}$$
$$\times \cos\beta\, d\beta, \quad \nu = 0;\ 2,$$

where

$$L_0 = L/a = [r_0^2 + (1 + \xi)^2 - 2r_0(1 + \xi)\sin\theta\cos\beta]^{1/2},$$
$$r_0 = R/a.$$

Since in the examined range the inequality $\gamma\sin\theta \gg 1$ is satisfied, then the internal integral in (38) may be converted according to the formula (see [7])

(39)
$$\int_0^{2\pi} \frac{e^{-i\gamma L_0}}{L_0} \cos\beta\, d\beta = -\frac{\pi i}{\sqrt{r_0(1 + \xi)\sin\theta}} \{H_0^{(2)}(\gamma b) +$$
$$+ i(-1)^{\nu} H_0^{(2)}(\gamma d)\} + O[(\gamma\sin\theta)^{-3/2}], \quad \nu = 0;\ 2,$$

where
$$b = [r_0^2 + (1 + \xi)^2 - 2r_0(1 + \xi)\sin\theta]^{1/2},$$
$$d = [r_0^2 + (1 + \xi)^2 + 2r_0(1 + \xi)\sin\theta]^{1/2}.$$

Substituting into (38) the values of functions $u^{(\nu)}(\xi)$ from (30) and applying formula (39) we obtain

(40)
$$\Lambda_x^{(\nu)} = -\frac{i e^{-i\frac{\pi}{4}} \omega \mu \rho I}{4} \frac{\sqrt{\gamma}}{2\pi\sin\theta\, a} \frac{1}{r_0} \{U_0^{(\nu)}[Q_1(2,\ r_0,\ 0) +$$
$$+ iQ(2,\ r_0,\ 0 + \pi)] + iC[Q_2(r_0,\ 0) + iQ_2(r_0,\ 0 + \pi)] -$$
$$- i\delta_{\nu}[Q_1(1,\ r_0, 0) + iQ_1(1,\ r_0,\ 0 + \pi)]\},$$

where

(41)
$$Q_1(\sigma, r_0, 0) = \frac{1}{\pi} \int_0^\infty \frac{e^{-i\gamma(\xi+\sigma)}\sqrt{\sigma}}{\sqrt{\xi}(\xi+\sigma)} H_0^{(2)}(\gamma b)\, d\xi;$$

(42)
$$Q_2(\sigma, r_0, 0) = \frac{1}{\pi} \int_0^\infty \frac{e^{-i\gamma\xi}}{\sqrt{\xi}} H_0^{(2)}(\gamma b)\, d\xi.$$

The integral $Q_1(\sigma, r_0, \theta)$ was examined in detail in [7]. In the far zone (with $r_0 \to \infty$) the following asymptotic equation is valid:

(43)
$$Q_1(\sigma, r_0, \theta) = \frac{\sqrt{2}\, e^{-\frac{\pi}{4}}}{\sqrt{\pi\gamma}}\, \frac{e^{-i\gamma r_0}}{\sqrt{r_0}}\, e^{-i\gamma(\sigma-1)\sin\theta}\{1 - \\ - \Phi(\sqrt{i\gamma\sigma(1-\sin\theta)})\} + O(r_0^{-3/2}).$$

The integral $Q_2(r_0, \theta)$ is calculated in [8] and is equal to:

(44)
$$Q_2(r_0, \theta) = \frac{\sqrt{2}}{\sqrt{\pi\gamma}}\, e^{i\frac{\pi}{4}} e^{-i\gamma(r_0-1)\sin\theta}[1 - \Phi(\sqrt{i\gamma r_0(1-\sin\theta)})].$$

In the far zone (with $r_0 \to \infty$) expression (44) acquires the form

(45)
$$Q_2(r_0, 0) = \frac{\sqrt{2}}{\pi\gamma}\, \frac{e^{-i\gamma r_0}}{\sqrt{r_0}}\, \frac{e^{i\gamma\sin\theta}}{\sqrt{1-\sin\theta}} + O(r_0^{-3/2}).$$

Using relationships (43) and (45) and moving from components $A_x^{(0)}$ and $A_x^{(2)}$ in the Cartesian coordinate system to components A_φ and A_θ in the spherical system of coordinates we obtain:

$$(46) \qquad A_\varphi = \frac{\omega\mu}{4\pi} \frac{p}{a} \frac{e^{-i\gamma r_0}}{r_0} S_\varphi(\theta)\sin\varphi;$$

$$(47) \qquad A_\theta = \frac{\omega\mu}{4\pi} \frac{p}{a} \frac{e^{-i\gamma r_0}}{r_0} S_\theta(\theta)\cos\varphi,$$

where

$$(48) \qquad S_\varphi(\theta) = \frac{1}{\sqrt{\sin\theta}} \left\{ [U_0^{(2)} - U_0^{(0)}] F_1(\theta) + F_2(\theta) \right\};$$

$$S_\theta(\theta) = \frac{\cos\theta}{\sqrt{\sin\theta}} \left\{ [U_0^{(2)} + U_0^{(0)}] F_1(\theta) - F_2(\theta) + \right.$$

$$+ C \frac{2e^{-i\frac{\pi}{4}}}{\sqrt{\pi\gamma}} \left(i \frac{e^{i\gamma\sin\theta}}{\sqrt{1-\sin\theta}} - \frac{e^{-i\gamma\sin\theta}}{\sqrt{1+\sin\theta}} \right) \right\};$$

$$(49) \qquad F_1(\theta) = e^{-i\gamma\sin\theta}[1 - \Phi(\sqrt{i\,2\gamma(1-\sin\theta)})] +$$

$$+ i e^{i\gamma\sin\theta}[1 - \Phi(\sqrt{i\,2\gamma(1+\sin\theta)})];$$

$$F_2(\theta) = i[1 - \Phi(\sqrt{i\gamma(1-\sin\theta)})] - [1 - \Phi(\sqrt{i\gamma(1+\sin\theta)})].$$

The intensity of the secondary electrical field $\vec{E}_1$ in the far zone is connected with the vector potential $\vec{A}$ by the relationship $\vec{E}_1 = -i\omega\vec{A}$. Consequently, the components of the intensity vector of the total electrical field $\vec{E} = \vec{E}_1 + \vec{E}_1{}^0$ in the far zone in the range $\gamma\sin\theta \gg 1$ are equal respectively to:

$$(50) \qquad E_\varphi = - \frac{e^{-i\gamma r_0}}{r_0} \frac{i\gamma^2 p \sin\varphi}{4\pi a^3} [S_\varphi(\theta) - i];$$

$$(51) \qquad E_\theta = - \frac{e^{-i\gamma r_0}}{r_0} \frac{i\gamma^2 p \cos\varphi}{4\pi a^3} [S_\theta(\theta) + i].$$

Let us move to calculation of the field in the area adjacent to

axis z, in which case we shall limit ourselves to the examination of the far zone.

Assuming in (38) that $L_0 \approx r_0 - (1+\xi) \sin\theta\cos\beta$, and changing the order of integration we obtain

$$
(52) \qquad A_x^{(\nu)} = \frac{i e^{\frac{i\frac{\pi}{4}}{4}}}{4\pi} \frac{\omega \mu p \sqrt{\bar{\gamma}}}{\sqrt{2\pi} a} \frac{e^{-i\gamma r_0}}{r_0} \int_0^{2\pi} G^{(\nu)}(\beta) e^{i\gamma\sin\theta\cos\beta} \times
$$
$$
\cos\nu\beta\, d\beta, \quad \nu = 0;\ 2,
$$

where

$$
(53) \qquad G^{(\nu)}(\beta) = \int_0^{\infty} u^{(\nu)}(\xi)\, \sqrt{1+\xi}\, e^{i\gamma\xi\sin\theta\cos\beta}\, d\xi, \quad \nu = 0;\ 2.
$$

Integral (53) may be calculated asymptotically. Substituting into (53) the values of functions $u^{(\nu)}(\xi)$ from (30) and disregarding terms of the order $O\{[\gamma(1-\sin\theta)]^{-3/2}\}$, we obtain

$$
(54) \qquad G^{(\nu)}(\beta) = \frac{1}{\sqrt{2\pi\gamma}} \frac{e^{-i\frac{\pi}{4}}}{\sqrt{1-\sin\theta}\cos\beta} K^{(\nu)} + O\{[\gamma(1-\sin\theta)]^{-3/2}\}.
$$

where

$$
(55) \qquad \left.\begin{array}{l} K^{(0)} = -i U_0^{(0)} e^{-i2\gamma} + \sqrt{2} C - \sqrt{2} e^{-i\gamma} \\ K^{(2)} = -i U_0^{(2)} e^{-i2\gamma} + \sqrt{2} C \end{array}\right\}.
$$

Expanding $(1-\sin\theta\cos\beta)^{-1/2}$ into a series with respect to degrees $\sin\theta\cos\beta$ and limiting ourselves to the first three terms of the expansion, following term-by-term integration in formula (52), we arrive at the following expression:

$$(56) \qquad A_z^{(v)} = \frac{i\,\omega\mu\,p}{4\pi a} \; \frac{e^{-i\gamma r_0}}{r_0} K^{(v)} T^{(v)}(0), \quad v = 0; 2.$$

where

$$(57) \qquad T^{(0)}(\theta) = J_0(\gamma\sin\theta) + \frac{1}{2}\sin\theta J_1(\gamma\sin\theta) + $$
$$+ \frac{3}{16}\sin^2(\theta)\,[J_0(\gamma\sin\theta) - J_2(\gamma\sin\theta)];$$

$$T^{(2)}(\theta) = -J_2(\gamma\sin\theta) + \frac{1}{4}\sin\theta\,[J_1(\gamma\sin\theta) -$$
$$(58) \qquad - J_3(\gamma\sin\theta)] + \frac{3}{32}\sin^2\theta\,[J_0(\gamma\sin\theta) - 2J_2(\gamma\sin\theta) + J_4(\gamma\sin\theta)].$$

Here J_n - Bessel function of the first type of order n. Moving to components A_φ and A_θ in the spherical system of coordinates we obtain

$$(59) \qquad A_\varphi = \frac{i\,\omega\mu\,p}{4\pi a} \; \frac{e^{-i\gamma r_0}}{r_0} V_\varphi(0)\sin\varphi;$$

$$(60) \qquad A_\theta = \frac{i\,\omega\mu\,p}{4\pi a} \; \frac{e^{-i\gamma r_0}}{r_0} V_\theta(0)\cos\varphi.$$

where

(61) $$V_\varphi(\theta) = K^{(2)} T^{(2)}(0) - K^{(0)} T^{(0)}(0);$$

(62) $$V_\theta(0) = [K^{(2)} T^{(2)}(\theta) + K^{(0)} T^{(0)}(0)] \cos 0.$$

Consequently, the components of the intensity vector of the total electrical field in the far zone in the range $\gamma(1-\sin\theta) \gg 1$ are equal to:

(63) $$E_\varphi = \frac{e^{-i\gamma r_0}}{r_0} \frac{\gamma^2 p \sin\varphi}{4\pi a^3 \epsilon} [V_\varphi(0) - 1];$$

(64) $$E_\theta = \frac{e^{-i\gamma r_0}}{r_0} \frac{\gamma^2 p \cos\varphi}{4\pi a^3 \epsilon} [V_\theta(0) + 1].$$

Thus, the supplementary problem of excitation of an ideally conducting surface with a round opening by an elementary electrical vibrator located in the center of the opening is completely solved. Let us move on to analysis of the initial problem.

EXCITATION OF A DISK BY AN ELEMENTARY MAGNETIC VIBRATOR

The electromagnetic field created by an elementary magnetic vibrator (by a slot on one side) located in the center of an ideally conducting disk, may be found with the aid of the principle of

duality [2] using the obtained solution. In this case in the far zone in the range $\gamma\sin\theta \gg 1$ the intensity of the total electrical field $\vec{E}'$ is determined by the following expressions:

a) in the upper half space ($z>0$):

$$(65) \qquad E_\theta' = H_\varphi' \sqrt{\frac{\mu}{\varepsilon}} = -\frac{e^{-i\gamma r_0}}{r_0} \frac{i\,\gamma^2 m \sin\varphi}{4\pi a^3\varepsilon} [S_\varphi(0) - 2i].$$

$$(66) \qquad E_\varphi' = H_\theta' \sqrt{\frac{\mu}{\varepsilon}} = -\frac{e^{-i\gamma r_0}}{r_0} \frac{i\,\gamma^2 m \sin\varphi}{4\pi a^3\varepsilon} [S_\varphi(0) + 2i].$$

where m - moment of the vibrator;

b) in the lower half space ($z<0$):

$$(67) \qquad E_\theta' = H_\varphi' \sqrt{\frac{\mu}{\varepsilon}} = -\frac{e^{-i\gamma r_0}}{r_0} \frac{i\,\gamma^2 m \sin\varphi}{4\pi a^3\varepsilon} S_\varphi(0).$$

$$(68) \qquad E_\varphi' = H_\theta' \sqrt{\frac{\mu}{\varepsilon}} = \frac{e^{-i\gamma r_0}}{r_0} \frac{i\,\gamma^2 m \cos\varphi}{4\pi a^3\varepsilon} S_\theta(0).$$

Respectively, in the area adjacent to axis z (i.e., with satisfaction of inequality $\gamma(1-\sin\theta) \gg 1$), the field in the far zone is determined by the formulas:

a) in the upper half space:

(69)
$$E_\theta' = H_\varphi' \sqrt{\frac{\mu}{\epsilon}} = -\frac{e^{-i\gamma r_0}}{r_0} \frac{\gamma^2 m \sin\varphi}{4\pi a^3 \epsilon}[V_\varphi(0) - 2].$$

(70)
$$E_\varphi' = H_\theta' \sqrt{\frac{\mu}{\epsilon}} = \frac{e^{-i\gamma r_0}}{r_0} \frac{\gamma^2 m \cos\varphi}{4\pi a^3 \epsilon}[V_\theta(0) + 2];$$

b) in the lower half space:

(71)
$$E_\theta' = H_\varphi' \sqrt{\frac{\mu}{\epsilon}} = -\frac{e^{-i\gamma r_0}}{r_0} \frac{\gamma^2 m \sin\varphi}{4\pi a^3 \epsilon} V_\varphi(0),$$

(72)
$$E_\varphi' = H_\theta' \sqrt{\frac{\mu}{\epsilon}} = -\frac{e^{-i\gamma r_0}}{r_0} \frac{\gamma^2 m \cos\varphi}{4\pi a^3 \epsilon} V_\theta(0).$$

NUMERICAL RESULTS

For comparison of the obtained asymptotic expressions (65)-(72)
with the results of the strict solution [1] numerical calculations
were made for the case when $\gamma = 5$.

Fig. 2 shows the standardized radiation pattern of an elementary
magnetic vibrator located in the center of an ideally conducting disk
on the upper side of the disk corresponding to the plane $\psi = 90°$. The
solid line shows values E_θ' of the component, referred to the maximum
value of the modulus $|E_\theta'|$, taken from [1] (strict solution). The
dotted line shows analogous values calculated according to formulas
(65), (69), (71).

Fig. 3 shows the standardized radiation pattern in the plane $\phi=0°$. The solid line corresponds to the strict solution and the dotted line, to values calculated according to formulas (66), (68), (70), and (72).

Fig. 4 shows the standardized radiation pattern of an elementary magnetic vibrator located in the center of an ideally conducting disk, in the plane $\phi=90°$, calculated according to formulas (65), (67), (69), and (71) with $\gamma=1C$.

Fig. 5 shows the standardized radiation pattern in the plane $\phi=0°$, calculated according to formulas (66), (68), (70), and (72) with $\gamma=10$.

Figures 6 and 7 plot analogous patterns with $\gamma=15$. As the calculations show, formulas (65)-(72) cover the entire range of change of angle θ.

The obtained solution is more precise, the larger the value $\gamma=ka$. However, as the numerical calculations show, it satisfactorily conveys the character of radiation patterns even with such a relatively small value of γ as $\gamma=5$.

The obtained solution is suitable only when the magnetic

vibrator liss in the disk, however, the employed method makes it possible to obtain a solution also for the case of a magnetic vibrator elevated above the disk.

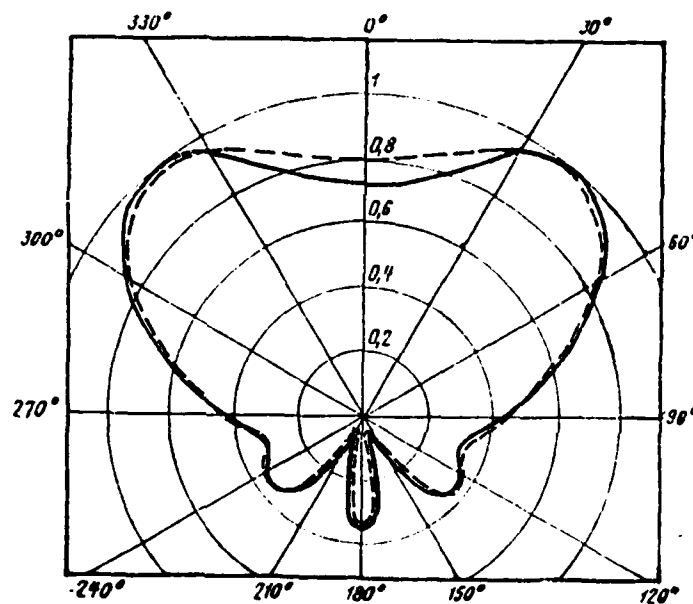In conclusion the authors wish to thank Professor G. Z. Ayzenberg and Docent L. S. Korol'kevich for discussion of this work and for valuable advice.
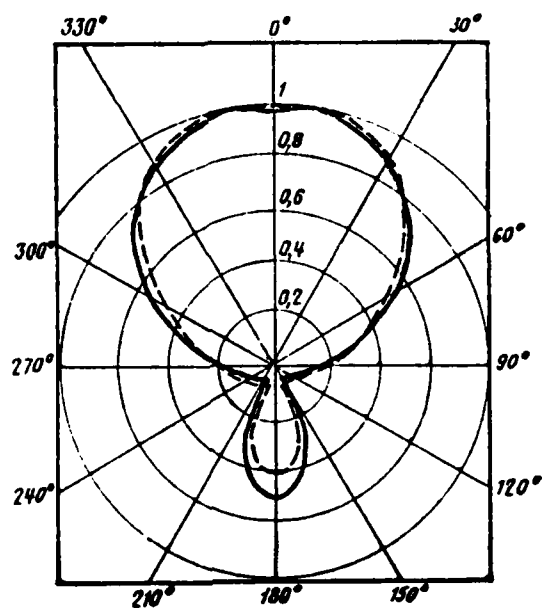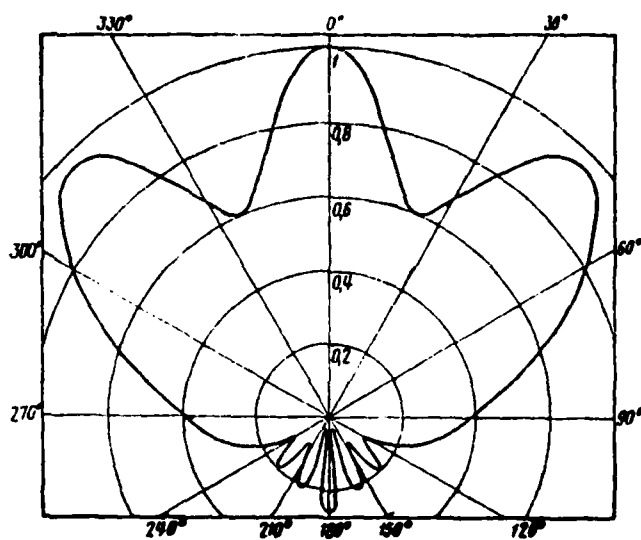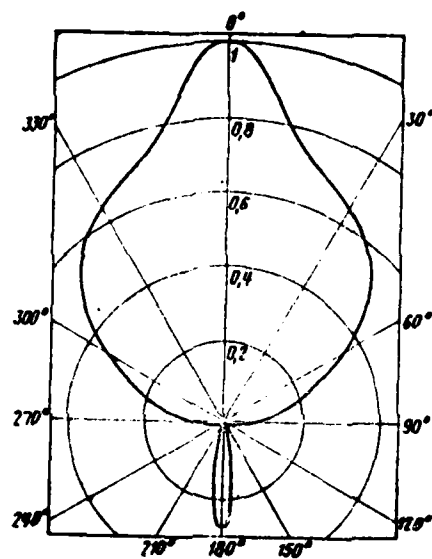

Fig. 2.
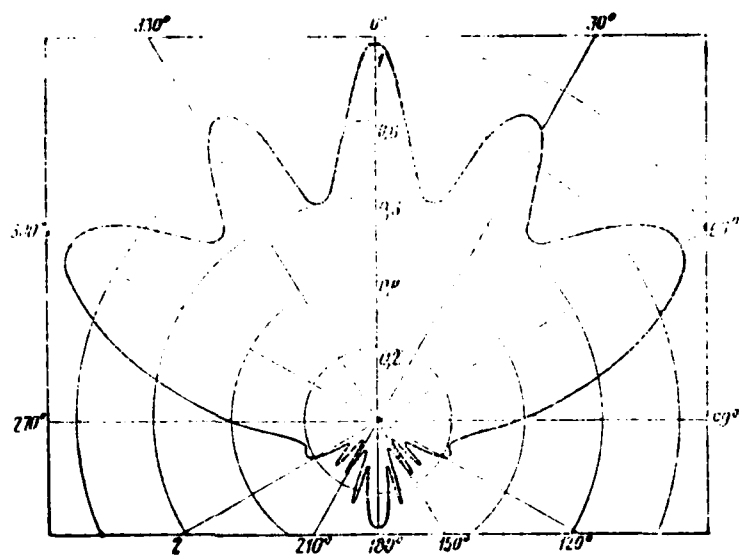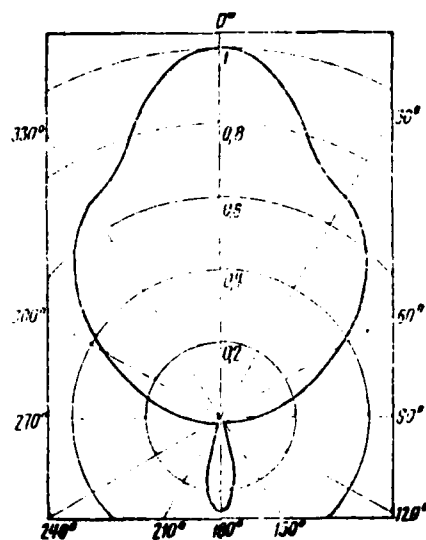
Fig. 3.

Fig. 4.

Fig. 5.

Fig. 6.

Fig. 7.

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